

Warm natural inflation

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Abstract

In warm inflation models there is the requirement of generating large dissipative couplings of the inflaton with radiation, while at the same time, not de-stabilising the flatness of the inflaton potential due to radiative corrections. One way to achieve this without fine tuning unrelated couplings is by supersymmetry. In this paper we show that if the inflaton and other light fields are Pseudo-Nambu-Goldstone Bosons then the radiative corrections to the potential are suppressed and the thermal corrections are small as long as the temperature is below the symmetry breaking scale. In such models it is possible to fulfill the contrary requirements of an inflaton potential which is stable under radiative corrections and the generation of a large dissipative coupling of the inflaton field with other light fields. We construct a warm inflation model which gives the observed CMB-anisotropy amplitude and spectral index where the symmetry breaking is at the GUT scale.

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1. INTRODUCTION

The cosmological theory of inflation [1] offers an explanation of the scale invariant horizon sized perturbations in the CMB observed by COBE [2] and WMAP [3–6]. A successful model of inflation requires the potential of the scalar field to be flat. A natural model for a flat potential is through Pseudo-Nambu-Goldstone bosons in a class of model called "natural inflation" [7–9]. The flatness of the inflaton potential in Natural inflation models is ensured by the fact that the PNGB couplings with other fields are suppressed by the symmetry breaking scale f and by making f large enough one can control the quantum corrections to the inflaton potential. One major problem of the Natural inflation models is that the spectral index of the inflaton perturbations is related to the symmetry breaking scale as $n_s = 1 - M_P^2/(8\pi f^2)$ and in order to be consistent with the WMAP [6] measurement $n_s = 0.963 \pm 0.014$, the symmetry breaking scale has to be close to the Planck scale [10]. In reference [11] it was pointed out that in warm inflation models [12–14] where there is a dissipative coupling between the inflaton and the radiation bath, one can have inflation with PNGB and the symmetry breaking scale could be lowered to the GUT scale $f \sim 10^{16}\text{GeV}$. In order to have a lower symmetry breaking scale in natural inflation models the dissipative coupling Γ of inflaton with radiation must be much larger than H .

Inflaton couplings with radiation fields through which one can generate large Γ are also expected to spoil the flatness of the inflaton potential due to radiative corrections[15, 16]. A solution for getting around the opposing constraints of obtaining a large dissipative coefficient but with small corrections to the inflaton potential, without fine tuning unrelated couplings, was obtained by invoking supersymmetry [17, 18]. Inflaton was coupled to heavy catalyst fields with masses larger than the temperature of the Universe and these fields in turn were coupled to light fields. The evolution of the inflaton would induce light particle production via the heavy catalyst fields. Since these heavy catalyst fields are expected to populate mostly their ground state, the quantum corrections associated with them could be canceled in super-symmetric models [13] however the temperature dependent dissipative damping terms are not canceled.

In this paper we study warm inflation caused by Pseudo-Nambu-Goldstone bosons as the inflaton. We study the finite temperature corrections to the potential. We show that the thermal corrections to the inflaton potential are small as long as the symmetry breaking scale $f \gg T$. We point out the possibility that if the light fields involved in the dissipative warm inflation are all PNGB's which arise in successive spontaneous symmetry breakings of the interactions of a heavy catalyst field then it is possible to generate a large dissipative couplings of the inflaton to radiation without destabilizing the inflaton potential. In the final section we show that taking the symmetry breaking near the GUT scale one can get warm inflation models with PNGB which satisfy all the observational constraints on the CMB-anisotropy spectrum from WMAP [6].

2. WARM INFLATION WITH PNGB

Natural inflation models use the PNGB potential of the form which arise from an explicit symmetry breaking,

$$V(\phi) = \Lambda(T)^4 \left(1 - \cos \left(\frac{\phi}{f(T)} \right) \right). \quad (1)$$

Here we assume that the spontaneous symmetry breaking scale f and the explicit symmetry breaking scale Λ are both dependent upon temperature through loop corrections. We will determine the finite temperature corrections to Λ and f taking an example of PNGB in a $SU(N)$ technicolor theory where we assume fermions condensate are formed giving rise to Goldstone bosons. Consider a theory with N flavors of fermions with an approximate chiral $SU_L(N) \times SU_R(N)$ symmetry. The axial part of the above symmetry is broken spontaneously, when the fermion condensate $\langle \bar{\psi}\psi \rangle$ attains a nonzero vev. The low energy dynamics is then given in terms of the matrix field $U(x) = \exp(2i\phi(x)/f)$ in terms of the $(N^2 - 1)$ Goldstone bosons $\phi(x)$. Here $\phi(x) = \phi^a(x)T^a$, $a = 1, N^2 - 1$ and T^a 's are the generators of $SU(N)$. We have shown in Appendix A that the temperature dependence of Λ and f are as follows ($\Lambda^4 \equiv -\frac{2m_f}{N} \langle \bar{\psi}\psi \rangle$ of Eqn. (A.3)),

$$\Lambda^4(T) = \Lambda_0^4 \left(1 - \frac{2(N^2 - 1)}{N} \frac{T^2}{12f^2} \right), \quad (2)$$

and

$$f(T) = f \left(1 - N \frac{T^2}{24f^2} \right) \quad (3)$$

to the leading order in T/f .

The dynamics of inflaton field is governed by the equation

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'(\phi, T) = 0. \quad (4)$$

Here over-dots represent derivative w.r.t t and \prime denotes differentiation with respect to ϕ and Γ is the damping term. The total energy density and pressure of the system is given by

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi, T) + Ts, \quad (5)$$

$$p = \frac{\dot{\phi}^2}{2} - V(\phi, T). \quad (6)$$

Here s is the entropy density of the system that is given by the derivative of the potential with respect to temperature. The Friedmann equation for expansion and the energy-momentum conservation equation are given by

$$H^2 = \frac{8\pi}{3M_p^2} \rho, \quad (7)$$

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (8)$$

From above equations we get the energy conservation equation for the radiation as

$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2. \quad (9)$$

During warm inflation ρ_r remains nearly constant. Assuming a slow roll of the inflaton we neglect $\ddot{\phi}$ in the equation (4) and kinetic energy term in the total energy density. During inflation the potential energy of the inflaton field is dominant so we also can neglect the T 's term in the total energy density. So we get

$$\dot{\phi} = -\frac{V'}{3H + \Gamma}, \quad (10)$$

$$H^2 = \frac{8\pi}{3M_p^2} V. \quad (11)$$

The slow roll parameters are defined as

$$\begin{aligned} \epsilon &= \frac{M_p^2}{16\pi} \left(\frac{V'}{V} \right)^2, \quad \eta = \frac{M_p^2}{8\pi} \frac{V''}{V}, \\ \beta &= \frac{M_p^2}{8\pi} \frac{\Gamma' V'}{\Gamma V}, \quad \delta = \frac{M_p^2}{8\pi} \frac{T \frac{\partial V'}{\partial T}}{V'}, \quad c = \frac{T \frac{\partial \Gamma}{\partial T}}{\Gamma} \end{aligned} \quad (12)$$

and the ratio between the thermal damping and the damping due to expansion is given by

$$Q = \frac{\Gamma}{3H}. \quad (13)$$

Here three extra slow roll parameters appear because of ϕ dependence of damping term and temperature dependence of the potential and dissipation coefficient Γ . It was pointed out in [19] that these all parameters should be less than $1 + Q$ to realize warm inflation. Latter, a detailed stability analysis was made by Moss et al [20] and they showed that to have successful warm inflation the parameter β (b in [20]) must be less than $\frac{Q}{1+Q}$ and $|c| < 4$. This implies that thermal corrections to the inflaton potential must be suppressed. A detailed study of the perturbation spectra in warm inflation models has been done by [19–21] for constant as well as temperature dependent Γ . However, we use the results of [19, 20] to calculate the power spectrum and spectral index for curvature perturbations, which are given as

$$P_{\mathcal{R}} = \frac{\sqrt{\pi}}{2} \frac{H^3 T}{\dot{\phi}^2} (1 + Q)^{\frac{1}{2}}, \quad (14)$$

$$\begin{aligned} n_s &= 1 + \frac{1}{\Delta} \left[-\frac{3(2Q + 2 + 5Qc)(1 + Q)}{Q} \delta - \frac{9Q + 17 - 5c}{1 + Q} \epsilon \right. \\ &\quad \left. - \frac{9Q + 1}{1 + Q} \beta - \frac{3Qc - 6 - 6Q + 2c}{1 + Q} \eta \right], \end{aligned} \quad (15)$$

where $\Delta = 4(1 + Q) + (Q - 1)c$.

In the following sections we calculate the dissipation coefficient in a model where the inflaton and the light fields are PNGB fields which interact through a heavy catalyst field. We will compute the amplitude and spectral index of the curvature perturbation in this model and compare with the WMAP observations to fix the scale of the symmetry breaking.

3. DISSIPATION THROUGH GOLDSTONE-BOSONS

In our model, inflaton field ϕ transfers its energy to the field σ via intermediate fields χ . We assume that both the inflaton and σ fields are PNCB with spontaneous symmetry breaking scales f and f_σ respectively. We work in a high temperature regime where $f > T > m_\chi > m_\sigma, m_\phi$.

The interaction of φ with χ is described by the Lagrangian

$$\mathcal{L} = \frac{1}{f^2} (\partial_\mu \varphi) (\partial^\mu \varphi) \chi^2 \quad (16)$$

and the interaction of χ with σ fields is described by

$$\mathcal{L} = \frac{1}{f_\sigma^2} (\partial_\mu \sigma) (\partial^\mu \sigma) \chi^2. \quad (17)$$

Such interactions are similar to Goldstone boson baryon interaction term in chiral perturbation theory [22]. Integrating by parts, interaction (16) can be re-expressed as

$$\mathcal{L} = -\frac{1}{f^2} (\varphi \partial_\mu \partial^\mu \varphi \chi^2 + 2\varphi \partial^\mu \varphi \chi \partial_\mu \chi) \quad (18)$$

Here $\varphi = \phi + \phi_1$, ϕ representing the zero mode classical field. Hence the interaction term (19) with the classical background field can be expressed as

$$\mathcal{L} = -\frac{1}{f^2} (\phi \ddot{\phi} \chi^2 + 2\phi \dot{\phi} \dot{\chi} \chi + 2\phi \partial^\mu \phi_1 \chi \partial_\mu \chi) \quad (19)$$

In slow-roll regime the $\ddot{\phi}$ term can be neglected. The dissipation coefficient Γ can be derived from the effective equation of motion that for the above interaction can be given as [18]

$$\ddot{\phi}(t) + V'(\phi) - \frac{2\dot{\phi}}{f^2} \langle \dot{\chi} \chi \rangle - \frac{2}{f^2} \langle \partial^\mu \phi_1 \chi \partial_\mu \chi \rangle = 0, \quad (20)$$

where $\langle \dots \rangle$ represents ensemble averages with respect to an equilibrium (quantum or thermal) state. These field averages can be calculated using linear response theory and $\langle \dot{\chi} \chi \rangle$ can be written to the first order as

$$\langle \dot{\chi} \chi \rangle = \langle \dot{\chi} \chi \rangle_0 - i \left(-\frac{2}{f^2} \right) \int d^4 x' \theta(t - t') \left[\dot{\phi}(x') \phi(x') - \dot{\phi}(x) \phi(x) \right] \langle [\dot{\chi}(x') \chi(x'), \dot{\chi}(x) \chi(x)] \rangle. \quad (21)$$

Here $\langle \dot{\chi} \chi \rangle_0$ represents the correlation function evaluated at initial time t . In slow-roll regime we can take $\dot{\phi}(t)$ as nearly constant so the effective equation of motion (20) becomes

$$\ddot{\phi}(t) + V'(\phi) - \frac{2\dot{\phi}}{f^2} \left[\langle \dot{\chi} \chi \rangle_0 - i \left(-\frac{2\dot{\phi}(t)}{f^2} \right) \int d^4 x' \theta(t - t') (\phi(x') - \phi(x)) \langle [\dot{\chi}(x') \chi(x'), \dot{\chi}(x) \chi(x)] \rangle \right] = 0. \quad (22)$$

The non-local term $(\phi(x') - \phi(x))$ in the integrand of the above equation can be simplified by Taylor expanding $\phi(x')$ and the effective equation of motion becomes

$$\ddot{\phi}(t) + V'(\phi) - \frac{2\dot{\phi}}{f^2} \left[\langle \dot{\chi}\chi \rangle_0 + i \left(\frac{2\dot{\phi}(t)^2}{f^2} \right) \int d^4x' \theta(t-t') (t'-t) \langle [\dot{\chi}(x')\chi(x'), \dot{\chi}(x)\chi(x)] \rangle \right] = 0. \quad (23)$$

We see that the contribution to the dissipation coefficient Γ comes from the two coefficients of the $\dot{\phi}$ in Eq. (23). The first term is given by

$$\Gamma_1 = -\frac{2}{f^2} \langle \dot{\chi}\chi \rangle_0, \quad (24)$$

while the second term will be

$$\Gamma_2 = -i \left(\frac{4\dot{\phi}(t)^2}{f^4} \right) \int d^4x' \theta(t-t') (t'-t) \langle [\dot{\chi}(x')\chi(x'), \dot{\chi}(x)\chi(x)] \rangle \quad (25)$$

In terms of the spectral functions of the χ field, the first term becomes [23]

$$\Gamma_1 = -\frac{2}{f^2} i \int \frac{d^4p}{(2\pi)^4} i p_0 \rho_\chi(p_0, \vec{p}) n_\chi(p_0) \quad (26)$$

where n_χ is the distribution function for the χ fields and ρ_χ is the spectral function defined as as

$$\rho_\chi(p_0, p) = \frac{2Im\Sigma_\chi}{(p_0^2 - \omega_p^2)^2 + (Im\Sigma_\chi)^2} \quad (27)$$

where $\omega_p = \sqrt{p^2 + m_\chi^2}$. Using the relation $Im\Sigma_\chi = 2\omega_p\Gamma_\chi$ and substituting (27) in (26) we get

$$\Gamma_1 = \frac{2}{f^2} \int \frac{d^3p}{(2\pi)^3} \frac{dp_0}{2\pi} p_0 \left[\frac{4\omega_p\Gamma_\chi}{(p_0^2 - \omega_p^2)^2 + (2\omega_p\Gamma_\chi)^2} \right] n_\chi(p_0) \quad (28)$$

For a given temperature, the p_0 integral is dominated by the point ω_p , which lies close to the poles of the spectral function. So to evaluate p_0 integral the integrand can be expanded about $p_0 = \omega_k$ and we obtain

$$\Gamma_1 = \frac{1}{f^2} \int \frac{d^3p}{(2\pi)^3} n_\chi(\omega_p) \quad (29)$$

that gives

$$\Gamma_1 \sim \frac{T^3}{f^2}. \quad (30)$$

This term is not sufficient for the strong dissipation as $\frac{T}{f} \ll 1$. To get large dissipation we will consider the second term (25) which can be represented diagrammatically as Fig. 1. As this is the main contribution to the dissipation coefficient so we will leave the subscript 2 in this term and denote it by Γ .

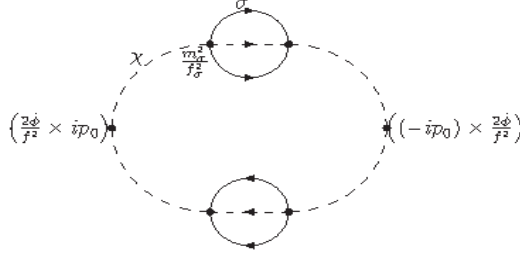


FIG. 1: The self energy diagram of φ which gives rise to the dissipation width Γ .

Again we can write Γ_2 in terms of spectral functions and we get [23],

$$\Gamma_2 = \Gamma = \frac{2}{T} \frac{4}{f^4} \dot{\phi}^2 \int \frac{d^4 p}{(2\pi)^4} p_0^2 \rho_\chi(p_0, \vec{p})^2 \times n_\chi (1 + n_\chi) \quad (31)$$

Again we can follow the similar steps as earlier to calculate Γ . Here after evaluating the energy integral we get

$$\Gamma = \frac{8}{T} \frac{\dot{\phi}^2}{f^4} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{4\Gamma_\chi} \times n_\chi(\omega_p) (1 + n_\chi(\omega_p)). \quad (32)$$

The relaxation time τ_χ for the χ can be calculated using the interaction (17). Since the σ field is also a PNGB it has derivative coupling to the intermediate field χ . We have used the method given in [24] to calculate the relaxation time. Here since the σ field is in thermal bath, the contribution to the relaxation time of the χ field will come from the thermal part of the free propagator of the internal lines. As it is evident from the finite temperature spectral function of the σ field (see Eq. (A10) of [24]), the derivative term in the interaction can be replaced by the mass of σ field using on-shell condition. Finally the relaxation time will be

$$\tau_\chi^{-1} = \Gamma_\chi = \frac{3}{8\pi} \frac{m_\sigma^4 T^2}{f_\sigma^4 \omega_p}. \quad (33)$$

Putting this expression for Γ_χ in (32) and redefining the integration variable $\beta\omega_p = x$ the dissipation coefficient will be

$$\Gamma = \frac{8}{3\pi} \frac{\dot{\phi}^2}{f^4} \frac{f_\sigma^4}{m_\sigma^4} T \int_{\beta m_\chi}^{\infty} x^2 \sqrt{(x^2 - (\beta m_\chi)^2)} \frac{e^x}{(e^x - 1)^2} dx. \quad (34)$$

Now in the regime $T > m_\chi$ the thermal integral can be evaluated numerically and it gives a factor of 7. Hence the dissipation coefficient will be

$$\Gamma = \frac{56}{3\pi} \frac{\dot{\phi}^2}{f^4} \frac{f_\sigma^4}{m_\sigma^4} T \quad (35)$$

We assume that ϕ and σ are generated from successive symmetry breaking such that $f > f_\sigma$. The PNGB masses m and m_σ are related to the symmetry breaking scales f and f_σ

respectively and some explicit symmetry breaking term in the Lagrangian like the fermions mass terms in a $SU(N)$ chiral symmetry breaking model. We will assume for simplicity that the explicit symmetry breaking scale for both the PNGB are of the same magnitude Λ . Then from the Gell-Mann-Okubo-Rienner relation we have $m^2 f^2 = m_\sigma^2 f_\sigma^2 = \Lambda^4$. Using the GOR relation and denoting $c_\phi = (f_\sigma/f)$ the expression for Γ in terms of c_ϕ can be written as

$$\Gamma = \frac{56}{3\pi} \frac{\dot{\phi}^2 c_\phi^8 f^4}{\Lambda^8} T. \quad (36)$$

There will be thermal corrections to the effective potential of ϕ due to the χ loop that can be given as [25]

$$V_{th} = \frac{m^2 \dot{\phi}^2}{8\pi f^4} \left(\frac{T}{m_\chi} \right) \phi^2. \quad (37)$$

With the choice of the parameters required to satisfy CMB observations (see the next sections) these corrections will be of the order of $\frac{m^2 \dot{\phi}^2}{8\pi f^4} \left(\frac{T}{m_\chi} \right) \sim 10^{-8} GeV^2$ which are much smaller than the mass term of the inflaton i.e $\sim 10^{16} GeV^2$.

In the next section we will use this expression in the formula (14) and (15) for the power spectrum amplitude and spectral index and fix the value of f which gives the correct CMB spectrum.

4. OBSERVATIONAL CONSTRAINTS

The PNGB potential parameters and warm inflation parameters can be constrained by using observational bounds on amplitude and spectral index of primordial perturbations.

Using expression (36) for Γ , the inflaton equation of motion (4) can be rewritten as

$$\ddot{\phi} + \left(3H + \frac{56}{3\pi} \frac{\dot{\phi}^2 c_\phi^8 f^4}{\Lambda^8} T \right) \dot{\phi} + V'(\phi) = 0. \quad (38)$$

To determine $\dot{\phi}$ we assume slow-roll approximation and we get

$$\dot{\phi} = - \left(\frac{3\pi V' \Lambda^8}{56 c_\phi^8 f^4 T} \right)^{1/3} \quad (39)$$

We use this value of $\dot{\phi}$ in the rest of the numerical calculations and we get $\Gamma \sim 10^{12} GeV$, which satisfies the conditions $\Gamma^2 \gg V''(\phi) \sim 10^{17} GeV^2$ and $\Gamma \gg H \sim 10^6 GeV$. This justifies our assumptions used in Eq. (39).

Assuming the amount of radiation produced by dissipation is nearly equal to the radiation diluted due to expansion, the radiation density is given as

$$4H\rho_r = \Gamma \dot{\phi}^2. \quad (40)$$

Using $\rho_r = \frac{\pi^2}{30} g_\star T^4$ and (35), (39) the temperature of the thermal bath is given as

$$T = \left(\frac{15}{2\pi^2 g_\star} \right)^{3/13} \left(\frac{3\pi}{56} \right)^{1/13} \left(\frac{\Lambda^8 (V'(\phi))^4}{H^3 c_\phi^8 f^4} \right)^{1/13}. \quad (41)$$

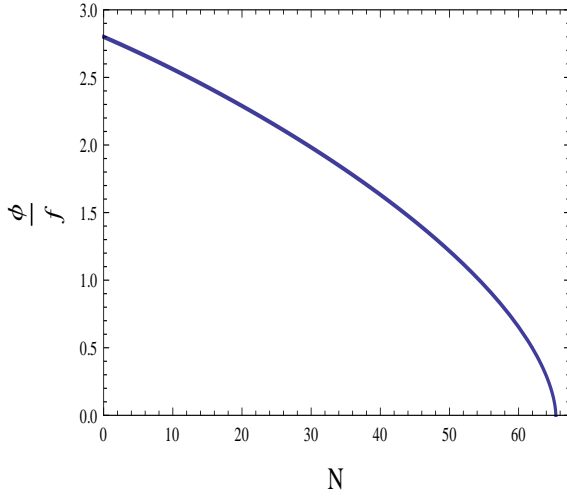


FIG. 2: Evolution of inflation w.r.t efolds.

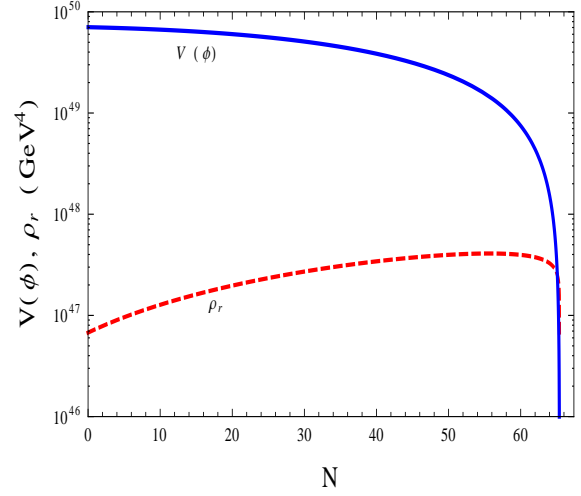


FIG. 3: Radiation density and potential energy w.r.t efolds

We have solved equation (38) and $\dot{H} = -4\pi G \left(\dot{\phi}^2 + \frac{4}{3}\rho_r \right)$ numerically using e-foldings $N = d \ln a$ as an independent variable. To solve these equation we take the values of the parameters that satisfy the observational constraints on A_s and n_s . We take $c_\phi = 0.145$, $f = 1.29 \times 10^{16} \text{ GeV}$ and $\Lambda = 2.45 \times 10^{12} \text{ GeV}$. The evolution of inflaton field ϕ , radiation density ρ_r and potential $V(\phi)$ is shown in Fig. 2 and 3. As depicted in Fig. 3, inflation ends when radiation density becomes equal to the potential energy of the inflaton field and we enter in a radiation dominated phase. With this choice of parameters, we can have sufficient number of e-foldings required to solve the horizon problem.

We have three parameters c_ϕ , f and Λ in this model that can be constrained from observations. In Fig.4, we have plotted the power spectrum and spectral index by varying f and Λ and using (36) for Γ and (41) for T . It is clear from the figure that the symmetry breaking scale is at the GUT scale to satisfy the observation constraints. From this figure we can see that for $A_s = 2.38 \times 10^{-9}$ and $n_s = 0.959$, which are within 1σ , $f = 1.29 \times 10^{16} \text{ GeV}$ and $\Lambda = 2.45 \times 10^{12} \text{ GeV}$. So for $\phi = 2.8 f$, we get the values for other parameters using Eq (36), (41), (39), (11) and we find that $\Gamma = 1.66 \times 10^{12} \text{ GeV}$, $T = 2.09 \times 10^{11} \text{ GeV}$ and $H = 1.99 \times 10^6 \text{ GeV}$. The ratio of the dissipation coefficient and Hubble constant $Q \sim 10^5$ is quite large while the thermal corrections to the inflaton potential are suppressed as T/f .

5. CONCLUSIONS

We have shown that it is possible to achieve natural inflation at the GUT scale $f \sim 10^{16} \text{ GeV}$ in warm inflation models if the dissipative coupling of the inflatons is large $\Gamma \sim 10^{12} \text{ GeV}$. We compute the thermal corrections to the PNCB potential and show that the corrections are small as long as $T \ll f$. We also show that a large dissipative coupling can be achieved if the both the inflaton and a light radiation field are PNCB's which arise in successive spontaneous symmetry breakings and which couple to a heavy catalyst field.

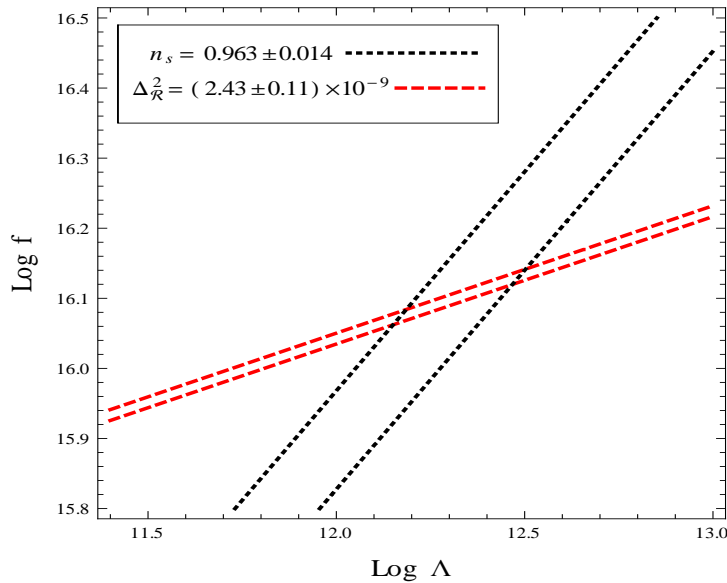


FIG. 4: The allowed range of $f(\text{GeV})$ and $\Lambda(\text{GeV})$ using the WMAP 7 constraints on amplitude $\Delta_{\mathcal{R}}^2$ and spectral index n_s of curvature perturbations.

Thus PNGB model of warm inflation is another class of models besides supersymmetry where one can naturally fulfill requirements of generating a large dissipative coupling without destabilizing the inflaton potential by thermal and quantum corrections.

It has been shown recently [26, 27] that in a model of inflation with PNGB there is a non-gaussianity due to the coupling of the pseudo-scalar with gauge bosons. In models such as ours where $f \ll M_P$ the non-gaussianity expected to be large and this could be observed in the forthcoming Planck [28] observations.

6. ACKNOWLEDGEMENT

We thank the referee for useful suggestions.

Appendix: Thermal corrections to the PNGB potential

Consider a theory with N flavors of fermions with an approximate chiral $SU_L(N) \times SU_R(N)$ symmetry. The axial part of the above symmetry is broken spontaneously, when the fermion condensate $\langle \bar{\psi}\psi \rangle$ attains a nonzero vev. The low energy dynamics is then given in terms of the matrix field $U(x) = \exp(2i\phi(x)/f)$ in terms of the $(N^2 - 1)$ Goldstone bosons $\phi(x)$. Here $\phi(x) = \phi^a(x)T^a$, $a = 1, N^2 - 1$ and T^a 's are the generators of $SU(N)$. Then the most general chirally invariant effective Lagrangian density with minimal number of derivatives given as[29]

$$\mathcal{L}_{eff} = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \quad (\text{A.1})$$

with f as the corresponding 'pion' decay constant. An explicit symmetry breaking gives rise to finite masses of the Goldstone 'pions' and is given by

$$\mathcal{L}_{sb} = \frac{-\langle\bar{\psi}\psi\rangle}{2N} Tr(MU^\dagger + UM^\dagger) \quad (\text{A.2})$$

where M is the fermion mass matrix which breaks the chiral symmetry explicitly. In the following we shall also assume the masses of the fermions to be the same m_f for all the flavors i.e $M = m_f I$ in which case the symmetry breaking term can be written explicitly in terms of the PNGB fields as

$$\mathcal{L}_{sb} = \frac{-\langle\bar{\psi}\psi\rangle m_f}{N} Tr \cos\left(\frac{2\phi^a T^a}{f}\right) \quad (\text{A.3})$$

where summation over all the flavors is understood for the condensate.

An explicit evaluation of the PNGB potential (A.3) up to quadratic terms in the fields gives us the Gellmann-Oakes-Renner (GOR) relation relating the PNGB mass m with the explicit symmetry breaking scale m_f ,

$$m^2 f^2 = -\frac{2m_f}{N} \langle\bar{\psi}\psi\rangle. \quad (\text{A.4})$$

To calculate the temperature dependence of the condensate, we use the Feynman - Hellmann theorem, according to which the value of the condensate $\langle\bar{\psi}\psi\rangle_T$ at finite temperature is related to the derivative of the free energy density with respect to the symmetry breaking parameter m_f ,

$$\langle\bar{\psi}\psi\rangle_T = \langle\bar{\psi}\psi\rangle + \frac{\partial}{\partial m_f} \tilde{\Omega}(T). \quad (\text{A.5})$$

Assuming that the thermodynamic potential is dominated by the Goldstone modes, the free energy difference $\tilde{\Omega}(T) = \Omega(T) - \Omega(T=0)$ is given as

$$\tilde{\Omega}(T) = \frac{(N^2 - 1)T}{(2\pi)^3} \int d\mathbf{k} \ln(1 - \exp^{-E/T}) \quad (\text{A.6})$$

where, $E = \sqrt{\mathbf{k}^2 + m^2}$ is the single PNGB energy. Next, we may use the fact that $\frac{\partial m}{\partial m_f} = \frac{m}{2m_f}$ from the GOR relation Eq.(A.4) and, eliminate the condensate $\langle\bar{\psi}\psi\rangle$ in favor of m using the same equation to obtain from Eq.(A.5) and Eq.(A.6),

$$\langle\bar{\psi}\psi\rangle_T = \langle\bar{\psi}\psi\rangle_0 \left(1 - \frac{2(N^2 - 1)}{N} t_1\right), \quad (\text{A.7})$$

where,

$$t_1 = \frac{1}{f^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{2E} \frac{1}{\exp(-E/T)} \simeq \frac{T^2}{12f^2} \quad (\text{A.8})$$

In the last step we have evaluated the integral in the chiral limit $m \ll T$ [30].

Next, we proceed to calculate the leading temperature dependence of the masses of the Pseudo Goldstone bosons. This can be calculated in the same line as has been worked out

in Ref.[31] for two flavor case using Hartree approximation. The key quantity here is to calculate the pion self energy in the medium. To lowest order, the interaction part is given by collecting quartic order fields in the expansion of Eq.(A.1) and Eq.(A.2) and takes the form

$$\mathcal{L}_{int} = \frac{1}{f^2} \left[\frac{m^2}{3} Tr(\phi^4) + 4 \left\{ \frac{1}{4} Tr(\partial_\mu \phi^2)(\partial^\mu \phi^2) - \frac{1}{6} Tr(\partial_\mu \phi)(\partial^\mu \phi^3) - \frac{1}{6} Tr(\partial_\mu \phi^3)(\partial^\mu \phi) \right\} \right]. \quad (\text{A.9})$$

$$(\text{A.10})$$

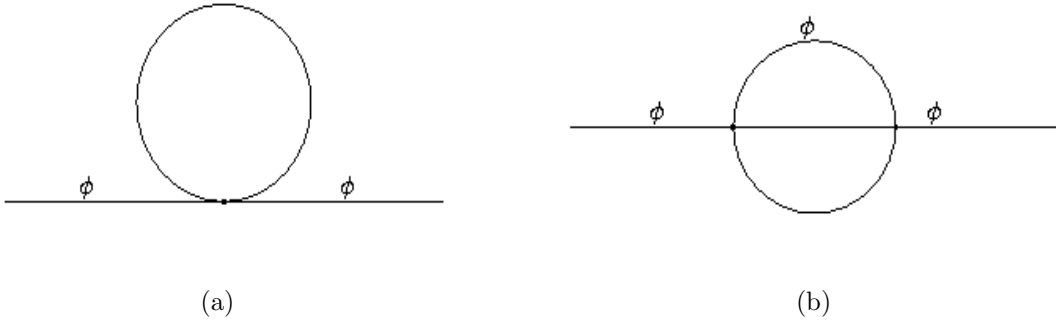


FIG. 5: *PNGB self energy*

With the above interaction term, the contribution to the self energy is given by the tadpole diagram and the sun set diagram shown in Fig. 5(a) and 5(b). The Hartree approximation corresponds to evaluating these diagrams with the full single particle Greens function, rather than the bare ones. Explicitly within Hartree approximation, the PNGB self energy is then given as [31]

$$\Pi(\omega, \mathbf{k}, T) = \left[-\frac{2m^2}{3N}(2N^2 - 3) + (\omega^2 - \mathbf{k}^2 + \tilde{m}^2) \frac{2N}{3} \right] t \quad (\text{A.11})$$

In the above, the first term in RHS originates from the term proportional to m^2 in the Lagrangian the interaction term in Eq. (A.10). The other terms originate from the derivative couplings in the Lagrangian Eq.(A.10) acting respectively on the external PNGB propagator $(\omega^2 - \mathbf{k}^2)$ and on the PNGB loop (\tilde{m}^2) . \tilde{m} is the in medium PNGB mass. Here we have used the trace relation for the $SU(N)$ generators $Tr(T^a T^c T^c T^b) = (N^2 - 1)/(4N)\delta^{ab}$ and $Tr(T^a T^c T^b T^c) = -1/(4N)\delta^{ab}$. further, we have here included the temperature dependent part of the propagator for the PNGB field and have discarded the vacuum contribution. The term t is the same as in Eq.(A.8) except the change $E \rightarrow \tilde{\omega} = \sqrt{(\tilde{m}^2 + \mathbf{k}^2)}$, denoting the in medium PNGB dispersion relation with the effective mass \tilde{m} . The effective mass satisfies the equation

$$\tilde{m}^2 = m^2 + \Pi(\tilde{m}, \mathbf{k} = \mathbf{0}, T) \quad (\text{A.12})$$

leading to

$$\tilde{m}^2 = \frac{1 - \frac{2}{3N}(2N^2 - 3)t_1}{1 - \frac{4N}{3}t_1} m^2 \quad (\text{A.13})$$

further noting the fact that the GOR relation is also valid at finite temperature[31], we obtain the temperature dependence of the the decay constant, using Eq.(A.7) and Eq.(A.13),as

$$f(T) = f(1 - N \frac{T^2}{24f^2}) \quad (\text{A.14})$$

to the leading order in T/f .

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